Experiment 2: Amplitude Modulation and Demodulation

Objective

Amplitude modulation (AM) is one of the simplest methods for modulating a sinusoidal carrier wave. The first use of AM for transmission of voice signals by radio can be traced to the beginning of the $20th$ century. Yet, AM continues to be used today in many communication applications including broadcast radio, aircraft VHF radios and two-way radios.

In this experiment you will generate AM signals, study their time- and frequency-domain characteristics, and measure their modulation indices. You will investigate the use of envelope and coherent detectors in demodulating AM signals.

Prelab Assignment

1. Let $m(t) = A_m \cos 2\pi f_m t$ be the single-tone modulating signal with $f_m = 1$ kHz. Consider the AM signal:

$$\begin{aligned}\varphi_{\text{AM}}(t) &= A_c \cos 2\pi f_c t + m(t) \cos 2\pi f_c t, \\ &= \big[A_c + A_m \cos 2\pi f_m t\big] \cos 2\pi f_c t. \end{aligned} \tag{1}$$

A_c is the carrier amplitude and $f_c = 10$ kHz is the carrier frequency.

(a) Plot $\varphi_{\text{AM}}(t)$. Assume $A_c > A_m$.

(b) Determine and plot $\Phi_{\text{AM}}(f) = \mathcal{F}\big[\varphi_{\text{AM}}(t)\big]$.

(c) Determine the modulation index μ of the AM signal $\varphi_{\text{AM}}(t)$.

(d) Determine the sideband power P_{sb}, the carrier power P_c and the power efficiency of the AM signal $\varphi_{\text{AM}}(t)$. How would you change the modulating signal $m(t)$ to maximize the power efficiency of the AM signal?

(e) Consider the envelope detector based AM demodulator shown in Figure (1). If the

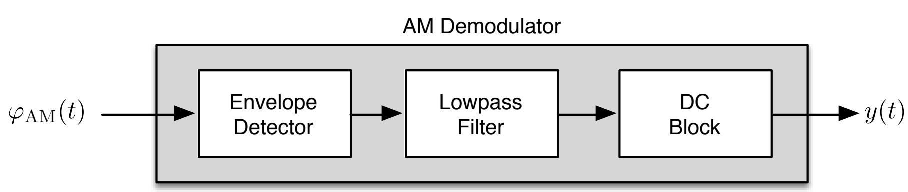


Figure 1: AM demodulation using an envelope detector.

lowpass filter has a cutoff frequency of approximately 1 kHz (slightly higher than f_m), sketch $y(t)$, output of the AM demodulator, when the modulation index of $\varphi_{\text{AM}}(t)$ is set to 50% and 120%.

2. Consider the *Hilbert transform* based envelope detector shown in Figure (2). This system presents an alternative to the traditional rectifier based envelope detector.

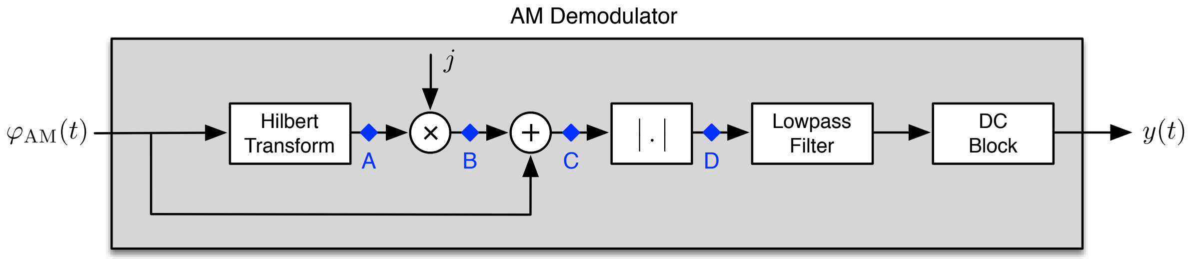


Figure 2: Hilbert transform based envelope detector.

The *Hilbert transform* block is described by the frequency response function:

$$H_h(f) = -j\text{sgn}(f) = \begin{cases} -j, & \text{if } f \geq 0; \\ +j, & \text{if } f < 0. \end{cases} \tag{2}$$

(a) Let the input to the envelope detector be the AM signal $\varphi_{\rm AM}(t)$ given in Equation (1). Using the result of Question **1.(b)**, sketch the spectra of the signals at test points A . . . C shown in Figure (2).

(b) Determine a time domain expression for the signal at test point D in Figure (2).

(c) Let $\varphi_{\rm AM}(t) = \big[A_c + m(t)\big] \cos 2\pi f_c t$ be an AM signal generated by an arbitrary modulating signal $m(t)$. Assume that $m(t)$ is band limited to B_m Hz, and $f_c \gg B_m$. Show that the envelope function $E(t)$ of the AM signal can be expressed as:

$$E(t) = \big|A_c + m(t)\big| = \sqrt{\varphi^2_{\rm AM}(t) + \varphi^2_{{\rm AM}-h}(t)} \tag{3}$$

where $\varphi_{{\rm AM}-h}(t)$ is the Hilbert transform of the AM signal $\varphi_{\rm AM}(t)$.

3. Download to oscilloscope/spectrum analyzer setup files **e2setupAB.scp** and **e2setupC.scp** from **[BlackBoard]** > **[Laboratory]** > **[Experiment 2]** to a USB drive.

Equipment

In this experiment you will use the following equipment and software:

- Agilent DSO-X 2002A digital storage oscilloscope with waveform generation and spectrum analyzer options.
- GW Instek GFG-8216A function generator.
- Hewlett Packard 33120A function/arbitrary waveform generator.
- Computer with Linux operating system.
- Matlab/Simulink 2014b.

Procedure

A. Characteristics of AM Signals

Part-A Setup

Function Generator 1–GFG-8216A **(FG1):** The settings are: **[Waveform:** sine**]**, **[Frequency:** 1 kHz**]** and **[Amplitude:** 5 V_{pp}**]**.

Function Generator 2–HP 33120A **(FG2):** The settings are: **[Frequency:** 10 kHz**]**, **[Amplitude:** 0.2 V_{pp}**]**, **[AM]** and **[Ext/Int Modulation]**. Consult your lab instructor for manual setup instructions. **Preset:** Required settings are stored in memory location **[1]** of the function generator. To access these settings press **[Recall]**, select memory location **[1]** and press **[Enter]**.

Oscilloscope/Spectrum Analyzer: Press **[Math]** > **[Operator:** FFT**]**. Use the following control settings: **[Source:** Channel 2 **]**, **[Span:** 20 kHz**]**, **[Center:** 10 kHz**]**, **[Window:** Rectangle**]**, **[Vertical Units:** V rms**]**. The last two control settings are accessible by pressing the **[More FFT]** softkey. **Preset:** Spectrum Analyzer settings used in Part-A are stored in the file e2setupAB.scp. To use the preset values: Press **[Save/Recall]** > **[Recall]** > **[Load from:** e2setupAB**]**.

Complete the connection diagram shown in Figure (3). Note that the output of FG1 must be connected to the **AM Modulation** input of FG2 which is located on its rear panel.

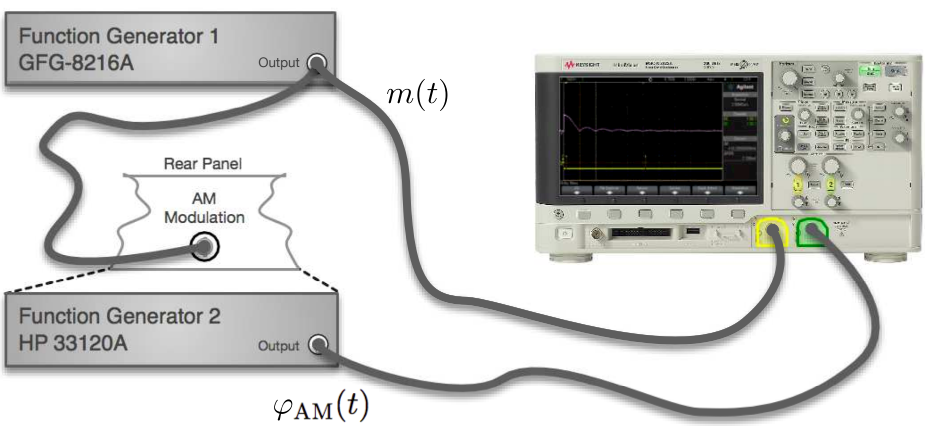


Figure 3: AM signal generation and **Part-A** connection diagram.

FG1 generates the single-tone modulating signal $m(t) = A'_m \cos 2\pi f_m t$ such that the output of FG2 is the AM signal:

$$\varphi_{\text{AM}}(t) = \big[A_c + Km(t)\big] \cos 2\pi f_c t \tag{4}$$

where $K = 0.2A_c$ is the gain of the built-in multiplier module in FG2. In this experiment we will use $A_c = 0.1$ V (corresponding to 0.2 V_{pp}). Thus, the single-tone modulated AM signal in Equation (4) can also be expressed as:

$$\varphi_{\text{AM}}(t) = \big[A_c + A_m \cos 2\pi f_m t\big] \cos 2\pi f_c t, \tag{5}$$

with $A_m = 0.02A'_m$.

Step A.1 Connect the output of FG1 (the single-tone modulating signal $m(t)$) to Channel 1 and the output of FG2 (amplitude modulated signal $\varphi_{\mathrm{AM}}(t)$) to Channel 2. Using the settings described in the **Part-A Setup** section, display $m(t)$, $\varphi_{\mathrm{AM}}(t)$ and the one-sided rms spectrum of the AM signal on the oscilloscope.

Step A.2 Change the amplitude and frequency of the modulating signal $m(t)$ and observe the corresponding changes in $\varphi_{\mathrm{AM}}(t)$ and its spectrum.

Step A.3 Observe how the envelope of the AM signal changes with changes in the amplitude of the modulating signal. Observe the phase shifts occurring in the AM signal at zero-crossing points of the envelope function when the AM signal is overmodulated, i.e., $\mu > 1$.

B. Modulation Index

Part-B Setup
Use the same setup as in **Part-A**.

Reset the amplitude and frequency of the modulating signal $m(t)$ and the carrier signal to values described in **Step A.1**.

Step B.1 Record/plot the AM signal $\varphi_{\mathrm{AM}}(t)$. Take measurements from the AM signal $\varphi_{\mathrm{AM}}(t)$ that would allow the calculation of the modulation index μ using *Method 1* described in the Appendix.

Step B.2 Switch the oscilloscope to the XY-mode (using the **[Horiz]** button). The display is known as the trapezoidal display, whereby the AM signal is shown versus the modulating signal. Record/plot the trapezoidal display. Take measurements from the trapezoidal display that would allow the calculation of the modulation index μ using *Method 2* described in the Appendix.

Step B.3 Display the one-sided rms spectrum of the AM signal (make sure that the vertical units for the spectrum analyzer is set to **[V rms]**). Measure and record the amplitudes and frequencies of the spectral components. Take readings from the spectrum of the AM signal that would allow the calculation of the modulation index μ using *Method 3* described in the Appendix.

Step B.4 Change the vertical units for the spectrum analyzer to **[dB]**. Measure and record the amplitudes and frequencies of the spectral components. Take measurements from the spectrum of the AM signal that would allow the calculation of the modulation index μ.

Step B.5 Adjust the amplitude of the modulating signal $m(t)$ to 4, 6, 8, 10, 12, 14 and 16 V_{pp} and repeats **Steps B.1–B.4**.

Problem B.1 Tabulate the measurements you took in **Steps B.1–B.5**. For each modulating signal amplitude, calculate the modulation index μ using all the methods described in the Appendix. Compare and comment on the results.

Problem B.2 If the modulating signal $m(t)$ is an arbitrary waveform, which of methods described in the Appendix can be used to measure the modulation index?

Problem B.3 For each modulating signal amplitude, use the spectral magnitude values you measured in **Step B.3/B.4** to calculate the sideband power P_{sb}, the carrier power P_c and the power efficiency of the AM signal. Plot the calculated P_{sb}, P_c and the power efficiency values vs. μ. Comment on the results.

Problem B.4 Consider the AM signal $\varphi_{\mathrm{AM}}(t) = \big[0.1 + A_m \cos 2\pi f_m t\big] \cos 2\pi f_c t$. Your objective is to maximize the power efficiency of the AM signal while being able to use an envelope detector for demodulation. Determine A_m.

C. Demodulation of AM Signals

In **Part-C** you will use two different envelope detectors to demodulate AM signals. The first detector uses a rectifier to extract the envelope of the AM signal, whereas the second detector uses the Hilbert transform based structure shown in Figure (2). Both detector structures are implemented in Matlab/Simulink.

The AM signal to be demodulated is generated as in **Part-A** of this experiment. This signal is fed to both the oscilloscope and the *sound-in port* of the computer. See Figure (4) for reference. Simulink models use the "sound" data from the *Audio Device* block, i.e., the sound-in port of the computer, as the input. Simulink models process this input and extract the modulating signal. You will monitor the outputs of the envelope detectors using the Simulink virtual instrument block *Scope*.

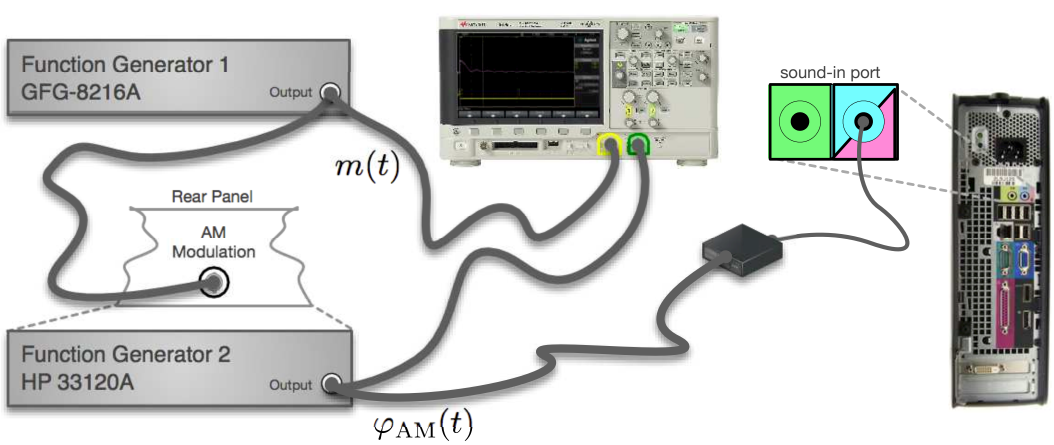


Figure 4: Part-C connection diagram.

Part-C Setup

Function Generator 1–GFG-8216A **(FG1):** The settings are: **[Waveform:** sine**]**, **[Frequency:** 100 Hz**]** and **[Amplitude:** 5 V_{pp}**]**.

Function Generator 2–HP 33120A **(FG2):** Same as in **Part-A/B**. The settings are: **[Frequency:** 10 kHz**]**, **[Amplitude:** 0.2 V_{pp}**]**, **[AM]** and **[Ext/Int Modulation]**. **Preset:** Required settings are stored in memory location **[1]** of the function generator. To access these settings press **[Recall]**, select memory location **[1]** and press **[Enter]**.

Oscilloscope: In **Part-C** we will use only the oscilloscope. Connect the modulating signal $m(t)$ (output of FG1) to Channel 1; connect the AM signal $\varphi_{\text{AM}}(t)$ (output of FG2) to Channel 2). **Preset:** Oscilloscope settings used in Part-C are stored in the file e2setupC.scp. To use the preset values: Press **[Save/Recall]** > **[Recall]** > **[Load from:** e2setupC**]**.

Step C.1 Generate an AM signal with carrier frequency $f_c = 10$ kHz as described in **Step A.1**. Adjust FG1 to output a single-tone modulating signal $m(t)$ with frequency to $f_m = 100$ Hz and amplitude 5 V_{pp} such that AM signal will have the modulation index $\mu \approx 0.5$. Connect the AM signal to the sound-in port of the computer and to Channel 1 of the oscilloscope; connect the modulating signal to Channel 2 of the oscilloscope. Use the connection diagram shown in Figure (4) for reference. Display the AM signal and the modulating signal on the oscilloscope.

Step C.2 Open and run the Simulink model **Exp2_EnvelopeDetectionRectifier.slx**. Observe the demodulated signal on the Simulink virtual scope. Compare the demodulated signal with modulating signal $m(t)$ and the envelope of the AM signal.

Step C.3 Gradually increase the amplitude of the modulating signal $m(t)$ and observe how the demodulated signal changes. Record/plot the demodulated signal and the AM signal when $\mu > 1$.

Step C.4 Open and run the Simulink model **Exp2_EnvelopeDetectionHilbert.slx**. Observe the demodulated signal on the Simulink virtual scope. Change the amplitude of the modulating signal $m(t)$ and observe the demodulated signal for the cases $\mu \leq 1$ and $\mu > 1$.

Problem C.1 AM radio broadcasting standards specify that AM radio stations must maintain 85–95% modulation with maximum modulation indices set to $\mu = 100\%$, $\mu_+ = 125\%$ and $\mu_- = 100\%$. Is it possible to use an envelope detector to demodulate an AM radio signal even though $\mu_+ = 125\%$? Explain.

Problem C.2 Which of the two envelope detector structures studied in **Part-C** would you use for installation in inexpensive radio receivers designed to receive AM broadcast radio signals?

Important : The next Simulink model uses a large buffer to store incoming signals. This results in high latency such that effects of changes in the AM signal appear on the Simulink virtual scope only after a rather long delay. Therefore, when you make changes to the AM signal, you are advised to stop and restart the Simulink model to observe changes in detector outputs.

Step C.5 Open the Simulink model **Exp2_EnvelopeAndCoherent.slx**. The model implements an envelope detector and a coherent detector to demodulate the AM signal. The coherent detector uses a *Phase-Locked-Loop* (PLL) tuned to the carrier frequency f_c to extract the carrier $\cos \omega_c t$ from the AM signal. In particular, we want to observe how well the two detectors function in demodulating AM signals with different modulation indices.

Adjust the amplitude of the modulating signal $m(t)$ to 5 V_{pp} to generate an AM signal with modulation index $\mu \approx 0.5$. Run the Sumulink model. Observe the demodulated signals at detector outputs; record/plot the signals displayed on the Simulink virtual scope.

Adjust the amplitude of the modulating signal $m(t)$ to 15 V_{pp} to generate an AM signal with modulation index $\mu \approx 1.5$. Restart the Simulink model. Observe the demodulated signals at detector outputs; record/plot the signals displayed on the Simulink virtual scope.

Problem C.3 Comment on the results you observed in **Step C.5**. How successful are the two detectors in demodulating AM signals with different modulation indices?

Problem C.4 The coherent detector in the Simulink model **Exp2_EnvelopeAndCoherent.slx** uses a PLL tuned to the carrier frequency f_c to extract the carrier from the AM signal. The output of the PLL (a sinusoid at carrier frequency f_c locked to the carrier in the AM signal) is in turn used to demodulate the AM signal. Assume that we replace the PLL with a local oscillator set to generate a sinusoid at carrier frequency f_c. Determine the output of this detector. Can it be used to demodulate the AM signal? Explain your answer.

Appendix: Modulation Index

Definitions and Discussions

Let $\varphi_{\text{AM}}(t)$ be amplitude modulated waveform:

$$\begin{aligned}\varphi_{\text{AM}}(t) &= m(t)\cos\omega_c t + A_c\cos\omega_c t, &\text{(A.1a)}\\ &= \big[\, A_c + m(t)\,\big]\cos\omega_c t, &\text{(A.1b)}\end{aligned}$$

with $\omega_c = 2\pi f_c$. The **modulation index** μ of an AM signal is defined as:

$$\mu = \frac{m_p}{A_c} = \frac{\max_t |m(t)|}{A_c}, \tag{A.2}$$

where m_p is the maximum absolute value of $m(t)$ and A_c is the carrier amplitude. Let $A(t)$ be the **amplitude function** of the AM signal:

$$A(t) = \big[A_c + m(t)\big], \tag{A.3}$$

and $E(t)$ be the **envelope function**:

$$E(t) = \big|A_c + m(t)\big|. \tag{A.4}$$

The use of an envelope detector as an AM demodulator requires that the amplitude function $A(t)$ remains positive, i.e., $[A_c+m(t)] \geq 0$, at all times. If this happens to be the case, then the envelope function equals to the amplitude function: $E(t) = \big|A_c+m(t)\big| = [A_c+m(t)] = A(t)$. Under these conditions an envelope detector together with a DC blocker can easily extract the modulating signal $m(t)$ from the envelope function $E(t)$. Equation (A.2) implies that the condition "$[A_c+m(t)] \geq 0$ *at all times*" is equivalent to $0 \leq \mu \leq 1$.

If the positive and negative swings of $m(t)$ are symmetric in magnitude, then the definition of modulation index given in Equation (A.2) is sufficient. However, if $m(t)$ is not symmetric, we need to extend the definition of the modulation index. As a matter of fact, AM broadcasting standards impose separate conditions on the modulation index based on positive and negative swings of the amplitude function $A(t)$. Let

$$\begin{aligned}A_{\text{max}} &= \max_t A(t) = \max_t\big[A_c + m(t)\big], &\text{(A.5a)}\\ A_{\text{min}} &= \min_t A(t) = \min_t\big[A_c + m(t)\big]. &\text{(A.5b)}\end{aligned}$$

Figure (A.1) depicts the amplitude and envelope functions for an AM signal with $0 < \mu < 1$ and for an overmodulated AM signal ($\mu > 1$). Next, we introduce extended definitions of the modulation index:

$$\textbf{Modulation Index:} \qquad \mu = \frac{A_{\text{max}} - A_{\text{min}}}{2A_c}. \tag{A.6a}$$

$$\textbf{Positive Modulation Index:} \qquad \mu_+ = \frac{A_{\text{max}} - A_c}{A_c}. \tag{A.6b}$$

$$\textbf{Negative Modulation Index:} \qquad \mu_- = \frac{A_c - A_{\text{min}}}{A_c}. \tag{A.6c}$$

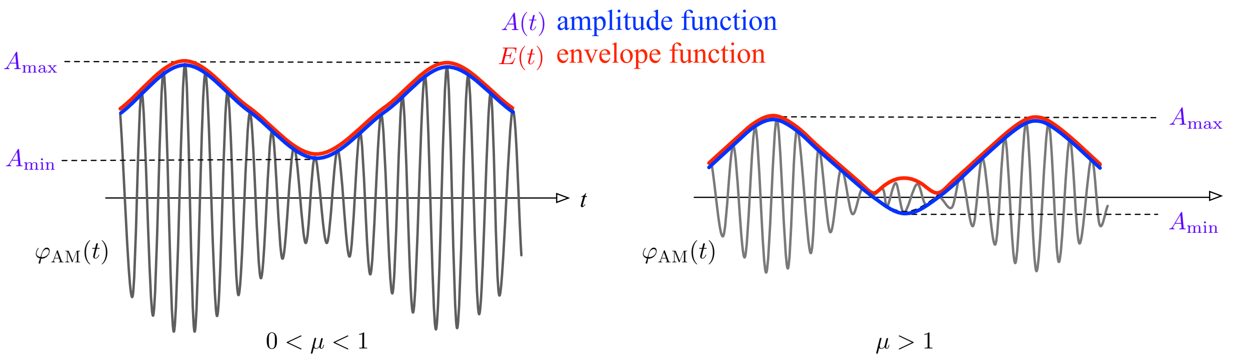


Figure A.1: Amplitude and envelope functions for different values of the modulation index μ.

Observe that if the positive and negative swings of $m(t)$ are equal in magnitude, i.e., if $|\max_t m(t)| = |\min_t m(t)|$, then $\mu = \mu_+ = \mu_-$.

Example 1: Single-Tone Modulation

Let $m(t) = A_m \cos\omega_m t$ be the single-tone modulating signal with $\omega_m = 2\pi f_m$. We generate the AM signal:

$$\varphi_{\mathrm{AM}}(t) = \left[\, A_c + A_m \cos\omega_m t \,\right] \, \cos\omega_c t. \tag{A.7}$$

We first evaluate the signal parameters:

$$m_p = \max_t |m(t)| = A_m, \qquad A_{\max} = A_c + A_m, \qquad A_{\min} = A_c - A_m.$$

Using Equation (A.2) we calculate:

$$\mu = \frac{m_p}{A_c} = \frac{A_m}{A_c}, \tag{A.8}$$

We also determine the extended definitions of the modulation index from Equations (A.6) as:

$$\mu = \frac{A_{\max} - A_{\min}}{2A_c} = \frac{[A_c + A_m] - [A_c - A_m]}{2A_c} = \frac{A_m}{A_c}, \tag{A.9a}$$

$$\mu_+ = \frac{A_{\max} - A_c}{A_c} = \frac{[A_c + A_m] - A_c}{A_c} = \frac{A_m}{A_c}, \tag{A.9b}$$

$$\mu_- = \frac{A_c - A_{\min}}{A_c} = \frac{A_c - [A_c - A_m]}{A_c} = \frac{A_m}{A_c}. \tag{A.9c}$$

In this example $m(t)$ is symmetric with respect to the horizontal axis so that the modulation indices evaluated using Equation (A.2) and Equation (A.6) are identical, and $\mu = \mu_+ = \mu_-$. Observe that in the case of single-tone modulation we can express the AM signal in Equation (A.7) in terms of the modulation index μ as:

$$\varphi_{\mathrm{AM}}(t) = \left[\, A_c + A_m \cos\omega_m t \,\right] \, \cos\omega_c t \tag{A.10a}$$

$$= A_c\left[1 + \frac{A_m}{A_c}\cos\omega_m t\right] \, \cos\omega_c t, \tag{A.10b}$$

$$= A_c\left[1 + \mu\cos\omega_m t\right] \, \cos\omega_c t. \tag{A.10c}$$

Example 2: Single-Tone Modulation from Part-A

In **Part-A** of this experiment we generated the AM signal

$$\varphi_{\text{AM}}(t) = \big[A_c + Km(t)\big]\cos\omega_c t \tag{A.11}$$

with $A_c = 0.1$ V, $K = 0.2$ and $m(t) = A'_m \cos\omega_m t = 2.5\cos\omega_m t$. Therefore,

$$\varphi_{\text{AM}}(t) = \big[0.1 + (0.02)(2.5)\cos\omega_m t\big]\cos\omega_c t, \tag{A.12a}$$
$$= 0.1\big[1 + 0.5\cos\omega_m t\big]\cos\omega_c t \tag{A.12b}$$

Comparing the AM signal we generated in **Part-A** with the general case in Equation (A.10c), we identify $\mu = 0.5$. Thus, the initial settings of the function generators in **Part-A** results in a single-tone modulated AM signal at 50% modulation.

Measuring the Modulation Index

Note: Discussion presented in this section assumes that the AM signal $\varphi_{\text{AM}}(t)$ is generated by a modulating signal $m(t)$ with equal magnitude positive and negative swings, i.e., $|\max_t m(t)| = |\min_t m(t)|$ such that $\mu = \mu_+ = \mu_-$. Furthermore, some of the methods of measuring the modulation index as presented in this Appendix can only be applied when $m(t)$ is a sinusoid.

Time Domain Methods

Method 1: Using the Amplitude Function

Connect the AM signal $\varphi_{\text{AM}}(t)$ to the oscilloscope and measure $A_{\max}$ and $A_{\min}$, the maximum and minimum values of the amplitude function $A(t)$, respectively. See Equation (A.5) and Figure (A.1). Once $A_{\max}$ and $A_{\min}$ are measured, determine the carrier amplitude A_c as:

$$A_c = \frac{1}{2}\big(A_{\max} + A_{\min}\big) \tag{A.13}$$

Modulation indices are then calculated using Equation (A.6).

Method 2: Using the Trapezoidal Display

Connect the AM signal $\varphi_{\text{AM}}(t)$ to Channel 1 of the oscilloscope, and connect the modulating signal $m(t)$ to Channel 2. Switch the horizontal control of the oscilloscope to XY-mode to display the trapezoid shown in in Figure (A.2). Measure α and β.

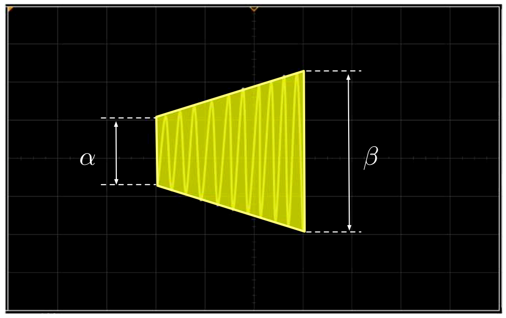


Figure A.2: Trapezoidal display used to measure the modulation index μ.

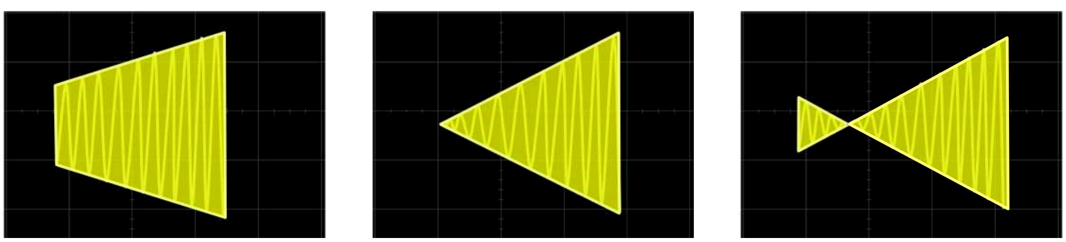
From the measurements we calculate:

$$A_{\min} = \frac{\alpha}{2}, \qquad A_{\max} = \frac{\beta}{2}, \qquad A_c = \frac{\beta + \alpha}{4}. \tag{A.14}$$

Using the definition given in Equation (A.6) we determine the modulation index as:

$$\mu = \frac{\beta - \alpha}{\beta + \alpha}. \tag{A.15}$$

The trapezoidal display makes it possible to quickly monitor the status of the AM signal and diagnose typical problems such as overmodulation. Figure (A.3) displays trapezoidal patterns corresponding to single-tone modulated AM signals with different modulation indices.



(a) (b) (c)

Figure A.3: Trapezoidal patterns for: **(a)** $\mu < 1$, **(b)** $\mu = 1$, and **(c)** $\mu > 1$ (overmodulation).

Frequency Domain Methods

Method 3: Using One-sided RMS Spectrum

Figure (A.4) shows the one-sided rms spectrum of the single-tone modulated AM signal in Equation (A.10c). The spectrum analyzer is set up to display the magnitudes of the spectral components on a linear scale (spectrum analyzer vertical units are set to V_{rms}).

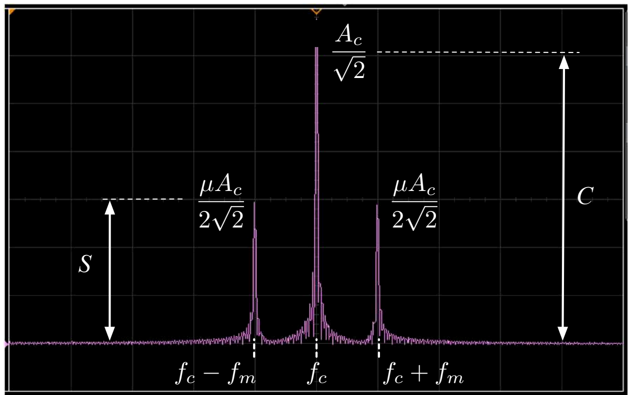


Figure A.4: One-sided rms spectrum of the single-tone modulated AM signal.

Let C be the magnitude of the carrier and S be the magnitude of the lower (or upper) sideband measured in V_{rms}. From the measured magnitude values of the carrier and sideband components we determine the modulation index μ as:

$$\mu = 2\,\frac{S}{C}. \tag{A.16}$$

If the modulation index is very low, the sideband magnitude may not be accurately determined when the spectrum analyzer displays the magnitudes of spectral components on a linear scale. In such cases we set the spectral analyzer vertical units to a logarithmic (dB) scale in order to magnify the sideband components.

Let C_{dB} and S_{dB} be the carrier and sideband magnitudes measured in dB. The counterpart of Equation (A.16) when the carrier and sideband components are measured in dB becomes:

$$20\log_{10}\frac{\mu}{2} = S_{dB} - C_{dB}, \tag{A.17}$$

We can easily solve Equation (A.17) to determine the modulation index μ.